ELECTROTECHNICAL APPROXIMATION FOR HEAVY-CURRENT RADIATING DISCHARGES

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Discharges in chambers with continuous (solid) electrodes are considered. It is shown that if the one-dimensional MHD approximation may be applied to these, then the far simpler electrotechnical approximation is applicable to the same accuracy. Calculations relating to exploding wires are described.

1. By passing a heavy current through a dense plasma it is not difficult to heat this to several electron volts and produce intense light emission. Such light sources have high electrical energy/radiation conversion efficiencies — sometimes over 50%; this explains why these devices have evoked major interest in the last few years. The corresponding problems are characterized by natural cylindrical symmetry. In the theoretical analysis and numerical simulation of such discharges, the one-dimensional MHD approximation is usually employed [1]. If, however, the MHD effects are slight, for calculating the electromagnetic field a simpler electrotechnical approximation may be used; this we shall now describe.

As in the earlier paper [1], we shall consider a discharge in the one-dimensional approximation, regarding it as cylindrically symmetrical. The motion of the plasma is described by the Euler equations with due allowance for the Lorentz forces:

$$\frac{\partial r}{\partial t} = v, \quad \rho r dr = dm,$$

$$\frac{\partial v}{\partial t} = -r\frac{\partial}{\partial m}(p + w) + \frac{F}{\rho}.$$

Here t is the time; ρ is the density, v is the velocity; r, m are the space and mass coordinates; p is the pressure; w is the pressure corresponding to artificial viscosity; and F is the Lorentz force.

The energy equation includes the Joule heating and the terms characterizing radiative heat transfer (the time derivative in both equations is Lagrangian):

$$\frac{\partial \varepsilon}{\partial t} = -\frac{\partial}{\partial m}(W + W_i) - (p + w)\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) + \frac{q}{\rho},$$

where ε is the internal energy of unit mass, q is the Joule heat, and W is the flow of energy due to electron-heat conduction:

$$W = -\varkappa_e \rho r^2 \frac{\partial T}{\partial m},$$

where T is the temperature; $\varkappa_e$ is the electron contribution to the thermal conductivity.

The radiation is assumed to be nonequilibrium. Radiative transfer is described by the multigroup kinetic equation

$$\vec{\Omega}\nabla I_k + \varkappa_k I_k = \frac{1}{\pi}\varkappa_k \sigma_k T^4,$$

$$\sigma_k = \int_{\nu_k}^{\nu_{k+1}} I_{eq}(\nu)\frac{d\nu}{T^4},$$

$$W_1 = \sum_{k=1}^{n} \varkappa_k \int \left(\vec{\Omega} \frac{\vec{r}}{r} \right) I_k (\vec{r}, \vec{\Omega}) \, d\vec{\Omega},$$

where $\varkappa_k$ is the average absorption coefficient for the group allowing for reradiation; $I_{eq}(T, \nu)$ is the spectral intensity of the equilibrium radiation.

Methods of calculating radiative transfer were given earlier [2]. The equations of gasdynamics were approximated by a purely implicit scheme, the gasdynamic terms of which were written in conservative form and the magnetic terms, in nonconservative form. Since the magnetic field is everywhere continuous, even in the shock waves, the conservative recording of the magnetic terms is optional. The energy balance was closely maintained in our calculations.

In the electrotechnical approximation the plasma column is a simple conductor, having a conductivity which varies over the cross section and characteristic proper values of the resistance, inductance, and capacity (the latter is negligibly small). The magnetic field at a specified point in the plasma is equal to

$$H(r) = 2I(r)/r; \quad I(r) = 2\pi \int_0^r r'j(r') \, dr'. \tag{1}$$

Here I(r) is the current flowing inside the specified radius.

The calculation for the resistance of the plasma column is obvious. The inductance is determined from

$$\frac{1}{2} L I^2 = \int \frac{1}{8\pi} H^2 dV,$$

where the integration is carried out over the whole region in which a field exists, i.e., in problems involving an outer return conductor, up to the return conductor, and in problems involving an inner return conductor, up to the outer boundary of the plasma column.

Using Eq. (1) we may express the inductance in terms of the distribution of the conductivity over the cross section. Here we must consider that the electric field strength is constant in the laboratory coordinate system and is related to the current density by

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{H}).$$

In the majority of problems the term $\vec{v} \times \vec{H}$ is a small correction. If we neglect this term, we obtain the following equation for the inductance of the plasma in a construction with an outer return conductor:

$$L_{pl} = 2 \cdot 10^{-9} l_{(cm)} \left\{ \int_0^{r_{\lim}} \frac{dr}{r} \left[\int_0^r \sigma(\xi) \xi d\xi \right]^2 \right\} \cdot \left[\int_0^{r_{\lim}} \sigma(\xi) \xi d\xi \right]^{-2}.$$

The total current in the circuit and the voltage on the electrodes are here determined from the solution of the electrotechnical equation

$$\frac{d}{dt} [(L_c + L_{pl}) I] + (R_c + R_{pl}) I + U_0 - \frac{1}{C_0} \int_0^t I \, dt = 0.$$

2. In experimental work the action is usually initiated with the aid of a spark discharge or the explosion of wires or foils. This creates a thin layer of plasma with a temperature of the order of several thousands of degrees. The exact values of the layer parameters are not determined in the experiments. In our present calculations the initiation is simulated by specifying the initial temperature in a layer 0.5 mm thick.

In order to discover the influence of the initiation conditions we executed several numerical calculations differing as regards the parameters of the heated layer. The calculations showed that for too low an initial temperature no discharge developed. Starting from a certain critical temperature ~0.3 eV the discharge develops normally, the progress of the action only depending on the initiation temperature at the very beginning. Ever after 0.2 μsec the difference between the various versions is erased. This is not hard to understand. On reducing the initial temperature the resistance of the layer increases. This leads to a great evolution of Joule heat and more rapid heating of the plasma; however, if the resistance is excessive, it actually opens the circuit and extinguishes the discharge. We note that an ignition temperature of around 3000-5000° is physically most reasonable for a layer thickness of the order of tenths of a mil-
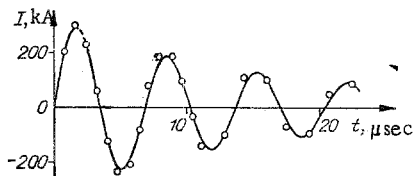
Fig. 1

limeter. However, in calculations based on the MHD approximation [1, 3] the computation could only be completed for an initial temperature of 1 eV or over; for a lower initial temperature the current suffered attenuation. In the electrotechnical approximation it is easy to calculate with physically reasonable parameters.

3. The developed stage of the discharge may be calculated equally accurately in either the MHD or the electrotechnical approximation (although the second approximation is much simpler). For illustration Fig. 1 presents current curves relating to one model problem. The points represent the result of the MHD calculation, and the line represents the electrotechnical approach.

Calculations for experimental constructions of small size revealed the excellent accuracy of the electrotechnical approximation. The calculations and physical analysis of the corresponding discharges will be published separately.

4. Let us consider the use of the electrotechnical approximation for wires and foils exploded by a rapid current pulse. These are certainly of independent interest [4]. In addition to this, such wires may serve for initiating heavy-current discharges.

At first glance it appears that, as a result of the high conductivity of the metal, the field diffusion time will be great, a skin layer will be formed, and the electrotechnical approximation will be invalid. However, this will only apply to weak currents, not leading to the explosion of the wire. On heating the wire to a temperature of the order of the melting point, its conductivity will fall rapidly, and the thickness of the skin layer will increase, normally becoming greater than the thickness of the wire itself. This means that the electrotechnical approximation will, in fact, be valid.

A number of consequences result. Firstly, of all these consequences were revealed in numerical calculations [5] but they may easily be understood from physical considerations. The heating of the wire, except at the very beginning of the process, is practically uniform over the whole cross section. The energy flows in so quickly that the wire is superheated above the boiling (evaporation) point and explodes simultaneously over its whole mass when the critical temperature is reached. The dense vapor so formed escapes into the ambient. When the density of the vapor falls below the so-called "metallization density" [6] the conductivity falls rapidly by three or four orders of magnitude, and the current is interrupted. The fall in density does not take place simultaneously over the whole cross section but in a release (unloading) wave passing from the outer boundary to the axis. Hence, the time of current interruption approximately equals the time of travel of the release wave

$$\tau = r_{lim}/c_{so},$$

where c_{so} is the velocity of sound. For a foil, half the thickness should be inserted into this equation. The time will be short, since the velocity of sound in the metal is high. It is not difficult to obtain an interruption time of under 0.1 μsec.

Since there is always an inductance in the circuit, the sharp current cut-off will lead to the development of excess voltages. It is thus not difficult to obtain voltages tens of times greater than those on the condenser battery discharging through the wire.

LITERATURE CITED

1. V. Ya. Gol'din, D. A. Gol'dina, G. V. Danilova, N. N. Kalitkin, L. V. Kuz'mina, S. P. Kurdyumov, A. F. Nikiforov, Yu. P. Popov, V. S. Rogov, V. B. Rozanov, A. A. Samarskii, V. B. Uvarov, L. S. Tsareva, and B. N. Chetverushkin, Study of Magnetic-Radiation Problems in Gasdynamics by Numerical Computer Methods [in Russian], Preprint No. 36, Izd. IPM, Moscow (1971).
2. V. Ya. Gol'din and B. N. Chetverushkin, "Methods of solving one-dimensional problems of radiative gasdynamics," Zh. Vyschit. Matem. i Matem. Fiz., No. 4, 990-1001 (1972).
3. Yu. P. Popov, "Completely conservative difference schemes and their application to the numerical study of problems in magnetohydrodynamics," Candidate's Dissertation, Institute of the Problems of Mechanics, Moscow (1971).

4. W. Chase, "Exploding wires," Usp. Fiz. Nauk, 85, No. 2, 381–386 (1965).
5. V. Ya. Gol'din and N. N. Kalitkin, Electrical Explosion of a Wire [in Russian], Izd. IPM, Moscow (1970).
6. V. A. Alekseev, "Measurement of the electrical conductivity and density of metals in the hypercritical state," Teplofiz. Vys. Temp., 8, No. 3, 641 (1970).